Using multiple representations in the classroom - The EdUmatics-Project

Caroline Bardini, Andreas Bauer, Ewald Bichler, Marie-Claire Combes, Jacques Salles, Hans-Georg Weigand,

University of Wuerzburg and University of Montpellier

Abstract. The EdUmatics-project (European Development for the Use of Mathematics Technology in Classrooms) aims to increase the integration of ICT in European mathematics classrooms. An online training course is constructed to provide learning and teaching material for in-service and pre-service secondary teachers. In this talk, we will give a short overview of the aims and methods of this project and we will present two activities. The first one illustrates what we call "teachers activities". It explores, using appropriate technological environments, the limits of a method used to determine the position of a curve with regard to its tangents. The other activity, which is to be conducted in the classroom, compares exponential and linear growth processes using interactive Geogebra applets or handheld devices. Results of empirical investigations carried out by French and German teachers about the later activities will also be examined.

Recent studies in Mathematics education show that despite many national and institutional actions within the EU aiming to integrate ICT into mathematics classrooms, such integration in secondary schools remains weak (e.g. Hoyles & Lagrange 2010). The rate of this integration increases slowly compared to the evolution of the technology. The huge diversity of ICT resources leaves teachers often unsure of which to use and when and how to use them. Studies also reveal that reasons for the slow integration of ICT in mathematics into classroom practice are deeply linked to the training strategies used. Approaches to training are sometimes unrelated to teachers' current classroom practices, being essentially based on the transmission of technological rather than pedagogical skills. Thus, there has been little impact on supporting teachers to make best use of new opportunities created by digital educational content and services.

The EdUmatics-Project

The European Development for the Use of Mathematics Technology in Classrooms (*EdUmatics*) project aims to increase thoughtful integration of ICT in European mathematics classrooms by building and disseminating an online training course for in-service and pre-service secondary teachers, in particular by providing high quality teaching material based on research and experience of the 20 partners involved (Drijvers & Weigand 2010). In using ICT, the partners are experts from ten universities and research institutes, together with ten secondary level schools across six European countries.

The project consists of five different "chapters" (called *modules*): 1. Starting to work with ICT, 2. From static to dynamic representations, 3. Constructing functions and models, 4. Using ICT in the classroom and 5. Multiple representations.

The Wuerzburg Group develops module 5 together with the IREM of Montpellier. This module deals with the use of multiple external representations (MER) in the classroom, the interrelationships between employed software and how to wisely use them in class. The module includes didactic considerations about the use of MER, methodical reflections on how to make thoughtful use of ICT, discussions concerning the theoretical background of MER in the learning of mathematics and ready-to-use classroom activities. Advantages and disadvantages, goals and difficulties of the use of multiple representations are also discussed.

The project will develop a research-based course, which aims to educate in-service and pre-service teachers to use new technologies in their mathematics classroom to maximise students' learning.

The course resources will become available through a multilingual European collaborative internet-based platform to include videos of classroom case studies, interactive applets, teaching materials, etc. This course also integrates face-to-face meetings (within each country), online-work, individual tasks and practical work in the classroom.

This paper will focus on the fifth module of the course: "Module 5: Multiple Representations". In the following, after having presented the outline of the module, we will examine two trends underlying the content of the module, namely the "teachers' activities" and the classroom activities. An illustrative example of both activities will be presented: "limits of a method" for the first trend and “growth processes” for the second one.

Multiple Representations (Module 5)

The structure of Module 5 is similar to the other four modules of the course, paying particular attention to the use of ICT to display mathematical objects in various ways. After an introductory part to show the possibilities of spreadsheets, dynamic geometry, graphing and CAS for mathematical representation, the module revolves around a three main topics: growth sequences, data and statistics and the derivative. These are primarily discussed in two parts, namely teachers’ activities and classroom activities, which will be described below.

Teachers' activities

Based on the large experience in working with teachers and in developing professional developments that aim to enhance mathematics education through technology, the Module 5 team believes that providing teachers with classroom activities that illustrate the use of environments supporting multiple representations, even when accompanied by substantial didactical and methodological elements, is not sufficient. Teachers also need to *reflect upon* their practice *before* they practice. For this purpose, several "teachers' activities" were included in the module, where teachers’ autonomy, considering their ICT choices, increases as they progress in the course.

Two kinds of activities were designed. Activities meant to familiarise teachers with one representation at the time (four in total, referred to as "basic competencies" in the course) were introduced at the earliest stages of the course. For most of them, the idea was to provide teachers with mathematical problems devised such that one particular representation would be especially helpful to make conjectures. The second kind of activities aimed to gradually introduce more than one representation that would potentially enrich the answer to the given mathematic question. The interplay between different representations, at the core of Module 5, is at the forefront of the later activities.

All teachers' activities are based upon the idea of an *a priori* analysis of classroom situations. Throughout the activities, teachers are asked to predict students' strategies used for solving the problem, reflect upon the technological environment they would use and argue on the advantages and disadvantages of different representations on offer. Hints on expected answers are given on teachers' demand (accessing the "expected answer" is possible through clicking on parts of the activities).

In order to illustrate the underlying principles of the teachers' activities, let us focus on the one called "Limits of a method", in which three representations (graphical, numerical and algebraic) are involved.

The aim of this activity was two-folded: not only has it been designed to put the limits of specific representations to solve the problem into evidence, but it also reveals the limits of a widely taught mathematical method used for examining the position of a curve with regard to its tangents, namely the method of studying the sign of an expression by factoring. This activity, as all teachers' activities, revolves around student tasks that begin with the following question:

> (C) designates the graphical representation of a function f chosen amongst common functions (such as “square”, “cubic”, “inverse” , “square root” , “polynomial” , “homographic” , “rational”) and T_0 the tangent to (C) at point M_0 of x-coordinate x_0.
>
> 1. What conjecture can you give for the position of (C) with regard to T_0?
>
> 2. Clarify the methods you have chosen in order to answer the previous question. What technological tools have you utilized?

Tasks addressed to teachers included naming the different environments and methods they think students would use to make conjectures and compare each of the environments. For the later question, for example, expected answers when analysing the advantages of the graphical environment are such as: "the visualization is more spontaneous and enhances possible conjectures. The graphic enables a more global view of the position of the two curves, while the spreadsheet limits the visualization to some rows only. Furthermore, the graphic enables one to make conjectures by anticipating the positions of the two curves for the portions not seen in the window".

In the following question, which was meant to investigate the status students attributed to the different environments, teachers were asked to predict the different environments and methods students would use to prove their conjectures.

3. Are the technological tools utilized to answer question 1 sufficient to prove the conjecture? If not, what would you suggest?

In fact, some students may realize that neither the graphical nor spreadsheet environment are sufficient to prove their conjectures about the position of (*C*) and T_0. In this case, it is likely they will think of using an algebraic method and therefore use CAS, the only environment that allows developing a rigorous proof. A possible method used for comparing two expressions consists in examining the sign of their difference, which may be done by factoring (when the sign is not evident). This method is particularly efficient for the common functions quoted in the subject, but not for other functions such as exponential. Question 4 of this activity (see below) was designed to reveal the limits of such a method.

Interestingly, an experiment conducted in a French grade 12 class, draws the attention on difficulties that have not been anticipated. In fact, some students, even if they have correctly envisaged to use an algebraic method and the CAS environment to prove their conjectures (which has been thought as the eventual main difficulty of this question), had great difficulties to interpret answers provided by CAS in order to match the conjecture, which was established from the graphical environment.

When considering the cubic function for this question, students frequently established the conjecture: "the graph of the cubic function is above the tangents on $\mathbb{R}^+$ and below on $\mathbb{R}^-$". With the help of CAS, students then find the factored expression and try to identify the positive and negative sign in such expression.

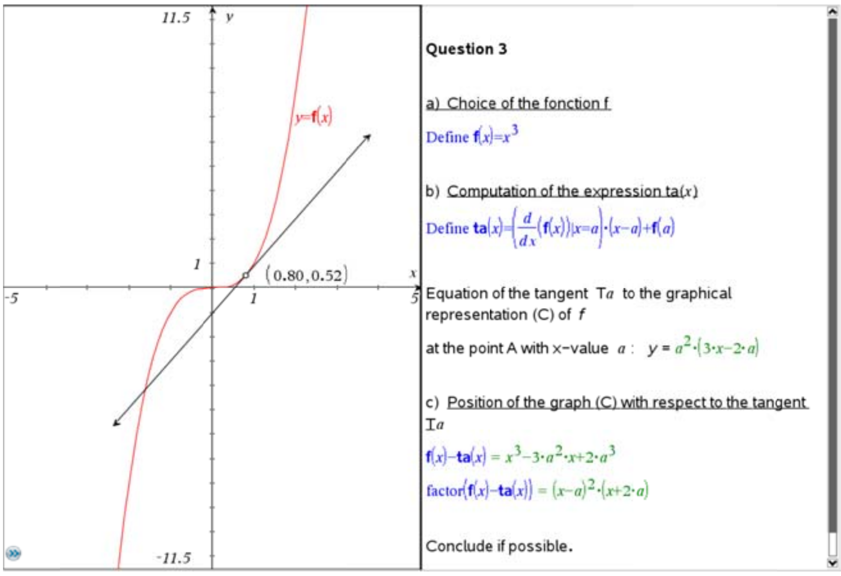


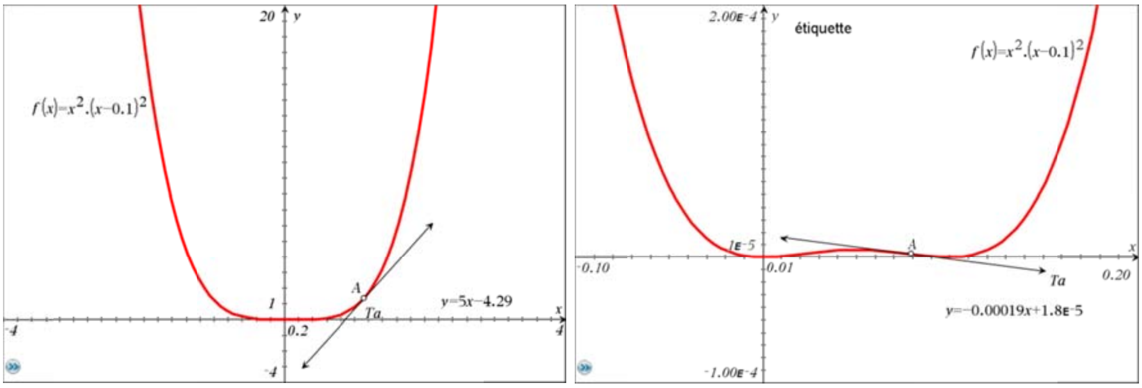
Fig. 1: Screenshot of the “Limits of a method” work sheet

While students easily recognised the positive sign of , they had trouble with interpreting the one of . And long discussions with the classroom teacher do not seem to suffice to students, whose final answers provide evidence that the interrelationship of both environments (graphical and algebraic) is far from been grasped by students: if and . The same difficulty arose in question 4, where the abscissa of the tangent point is numerically fixed.

Later in the activity, tasks addressed to the teachers relate to the following question:

4. Are the environments and methods utilized in the previous questions still relevant to answer questions 1 and 2 when the function is defined on ℝ by ?

Teachers are asked to predict students' answers and imagine a helpful action for students whose analysis may remain a bit shallow. This question is the occasion to alert teachers from some bias of the graphical environment. In fact, depending on the window chosen, the graphic visualization may hide some subtle characteristics of the function. In a "standard" window, for example, the curve seems to be always on the top of its tangents. Appropriate zooming is necessary, as shown in the figures below.



Issues about the interpretation of spreadsheets are also raised. In fact, when using "standards values" for the -value of the tangent point and, for this step, the spreadsheet provide a table of values for where all numbers have the same sign. To make the change of sign explicit, a specific value for (between 0 and 0.1) and for the step (strictly smaller than 0.1) is necessary.

The task ends with the analysis of , for which the method of factoring in order to examine the sign of the difference is not helpful anymore (the use of CAS shows that it is not possible to factor the expression); another method is needed.

Classroom activities - Growth processes

The overall aim of Module 5 is to foster the thoughtful use of multiple representations with ICT and to let teachers experience the relevant use of technologies in their classrooms on their own. To lower the entrance hurdle, we offer a number of pre-built classroom activities that are to be used directly in the participating teacher's classroom. These activities contain work sheets and suggestions for student tasks, but also methodical and didactic considerations on how to use these activities and especially ICT as a valuable supplement to the common classroom practice.

> In a quiz show, a candidate is to be asked up to ten questions. If he or she gives the wrong answer, the candidate must leave the show, but may keep the prize he or she won up to that question. Prior to being asked the first question, the candidate is given two alternative prize options from which he or she must choose one:
>
> Type 1: Each correctly answered question adds 100 €.
>
> Type 2: The first question correctly answered yields 20 €, each additional correct answer doubles the prize.
>
> a) Consider these two options! Which one would you choose spontaneously? Discuss with your neighbour.
> b) Give a value table, which displays the winnings depending on the number of correctly answered questions.
> c) Compare the two prize models and discuss which one might be preferred regarding certain circumstances.
> d) Would you change your mind if one of the amounts were different? Experiment with the applet on the right, then discuss your decision.

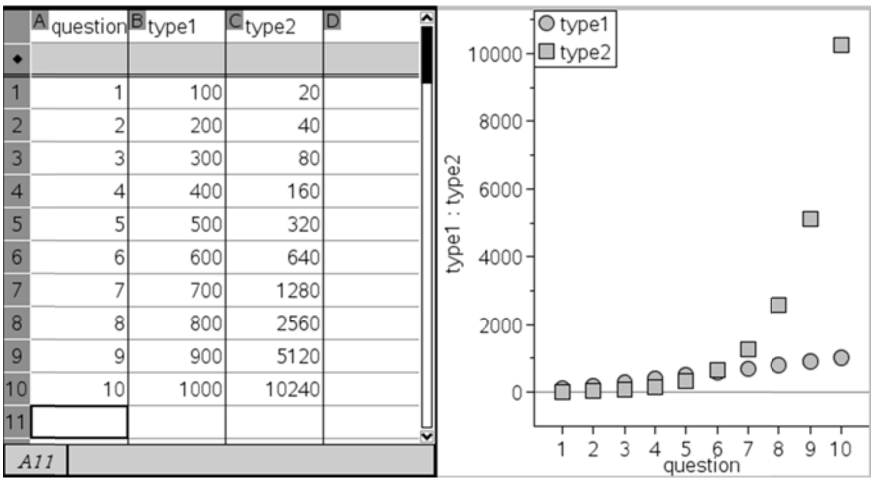
e) Would you change your mind if you were asked up to 15 questions instead of 10?

The aforementioned didactic considerations include a short overview of the mathematical background, a discussion of the topic's educational relevance, its prerequisites to be used in class and why this subject matter was chosen for multiple representations with ICT in the classroom. These student tasks constitute the heart of the classroom activities. The following example is taken from the activity "Quiz show", where students are to compare linear and exponential growth.

Growth processes and their model as growth sequences are good examples for classroom activities with multiple representations and the use of ICT. There are some competencies, which are related to this topic: dynamic experience of mathematics, working with different representations, emphasizing the relationship between algebra and geometry, etc. Depending on their complexity, growth sequences can also be used in any grade of secondary education, with linear growth in the lower and polynomial, exponential or logistic growth in the higher grades.

While symbolic explicit and recursive notations, discrete graphs and value tables are the sequences' basic mathematical representations, it is not easy to handle sequences as discrete objects on a symbolic level when using only paper and pencil. High school students are not confident with algorithmic knowledge concerning sequences. ICT is a tool to work with representations of discrete processes. We especially think about the use of spreadsheets, because the analogy between cells in spreadsheet programs and sequences' elements can be used in a beneficial way. When participating in the EdUmatics course, it is the teacher's task to integrate the classroom activity into his or her lessons, either by using pre-built programmes or applets. In our course, we use TI-Nspire programmes (see Fig. 3) and Geogebra applets. Both provide possibilities to use dynamically linked multiple representations (see also Guin 2005, Hegedus & Moreno-Armella 2008).

We intend to show teachers the benefit of certain representations, but also the inappropriateness of others and the use of multiple representations. This especially means to know about the role of these representations in the learning process and – as a consequence – in the teaching process. So, we first did a pilot investigation with university teacher students and students of grade 10 about the use of multiple representations with the "Quiz show" problem. We have been especially interested in the following research questions concerning



	A question	B type1	C type2	D
1	1	100	20	
2	2	200	40	
3	3	300	80	
4	4	400	160	
5	5	500	320	
6	6	600	640	
7	7	700	1280	
8	8	800	2560	
9	9	900	5120	
10	10	1000	10240	
11				

w"

the use of multiple representations:

1. Which representations do students use when solving the problem?
2. How do students use representations in their arguments?
3. How do the students use multiple representations and when do they switch representations?

The representations which students chose to solve the task, are highly dependent on their previous knowledge, in particular whether they used self-defined functions to model the situation or the spreadsheet. First classroom trials (The trials are not yet evaluated. We will present first results at the Portsmouth Conference.) using the TI-Nspire – in which students created their own representations – have shown that, as expected, most students preferred graphical representations for globally comparing the two types of growth, since it gives a more global view of the depicted growth sequences, especially – visually – giving a good impression of the exponential prize model. While value table and graph do not yield any additional information that cannot be read from the other representation as well, the representations' global or local views on the growth sequence support global or local approaches that complement and support each other (see Ainsworth 2006). For example, students switched in their argumentation to the spreadsheet view when a clear decision on the number of questions they considered themselves to be capable of answering correctly, giving arguments such as "winning type 2 is preferable over type 1, since I think I can probably answer at least six questions correctly and its winnings are higher with 640 € instead of 600 €". Experimenting with different starting values of the sequences in the task's part c) emphasized the link between the two representations, with ICT automatically translating changes in the spreadsheet view to the graph.

The idea of the project is that teachers, after trialing an activity in their own class, have the possibility to discuss their experiences with other participants of the EdUmatics course. Especially, they should reflect on the use of ICT of their students, the advantages and disadvantages they see and also on how to change the teaching units. To support this reflective process, methodical reflections at the end of the module give hints in advance and for the follow-up of the lesson. Moreover, the teachers are to discuss their results of the trials in relation with relevant research results, comparing them to pre-recorded, exemplary classroom episodes.

Conclusion

The study of a mathematical problem can often be enriched by investigating it from different points of views (e. g. Weigand & Bichler 2010). Multiple representations promote the emergence of conjectures at the same time it enriches them, assuring flowing links between them. However, using multiple representations requires teachers to work with their students in order to help them interpret information from different representations – see the comments on students' difficulties encountered in the "Limits of method" activity. The interrelation between representations is not self-evident, but takes a significant effort by the students. The use of interrelated software is a great way to help them with this task, but it is not sufficient if it does not come with competent guidance and instruction. The EdUmatics project provides training for teachers to qualify for this task, offering teacher activities to sensitise for important issues of using ICT, classroom activities that provide an easier access to new technologies in their own classrooms, underpinned by the current research in this area and with the possibility for reflection on their progress by communicating with other teachers participating in the course.

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